Testing Lorentz invariance in β decay

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Chapter 2

Concepts

This chapter discusses the consequences of LIV for β-decay observables of polarized nuclei. We first discuss the phenomenological decay rate. This is then related to the theory for LIV in β decay. We motivate and demonstrate the detection strategy of the experiment. An isotope is selected that can be conveniently spin-polarized.

2.1 General considerations

Taking into account the spin of the parent nucleus J, and the β particle properties of spin ($\boldsymbol{\sigma}_e$), energy (E_e), and direction (Ω_e), the SM differential decay rate for allowed transitions (the β particle and neutrino do not carry orbital angular momentum) is given by [17]

$$\begin{aligned}
&\omega_{\mathrm{SM}}(\langle \boldsymbol{J} \rangle, \boldsymbol{\sigma}_e | E_e, \Omega_e) dE_e d\Omega_e \\
&\qquad = \omega_0(E_e) dE_e d\Omega_e \times \left\{ 1 + \frac{\boldsymbol{p}_e}{E_e} \cdot \left(A \frac{\langle \boldsymbol{J} \rangle}{J} + G \boldsymbol{\sigma}_e \right) \right. \\
&\qquad \left. + \boldsymbol{\sigma}_e \cdot \left[N \frac{\langle \boldsymbol{J} \rangle}{J} + Q \frac{\boldsymbol{p}_e}{E_e + m_e} \left(\frac{\langle \boldsymbol{J} \rangle}{J} \cdot \frac{\boldsymbol{p}_e}{E_e} \right) \right] \right\} , \qquad (2.1)
\end{aligned}$$

where the coefficients A, G, N and Q are functions of the matrix elements depending on the nature of the transition; m_e and $\boldsymbol{p}_e$ are the electron mass and momentum, respectively, and $\langle \boldsymbol{J} \rangle / J$ is the nuclear polarization vector. The lowest level at which we can introduce LIV is then an explicit dependence on the direction $\boldsymbol{p}_e/E_e$, $\boldsymbol{J}$, and $\boldsymbol{\sigma}_e$, testing rotational invariance (see Fig. 2.1). Allowing also a boost dependence the terms in braces in Eq. (2.1) then read

$$1 + \xi_0 + \frac{\boldsymbol{p}_e}{E_e} \cdot \left(A \frac{\langle \boldsymbol{J} \rangle}{J} + G \boldsymbol{\sigma}_e + \boldsymbol{\xi}_1 \right) + \boldsymbol{\xi}_2 \cdot \frac{\langle \boldsymbol{J} \rangle}{J} + \boldsymbol{\sigma}_e \cdot \left(N \frac{\langle \boldsymbol{J} \rangle}{J} + \boldsymbol{\xi}_3 \right) . \quad (2.2)$$

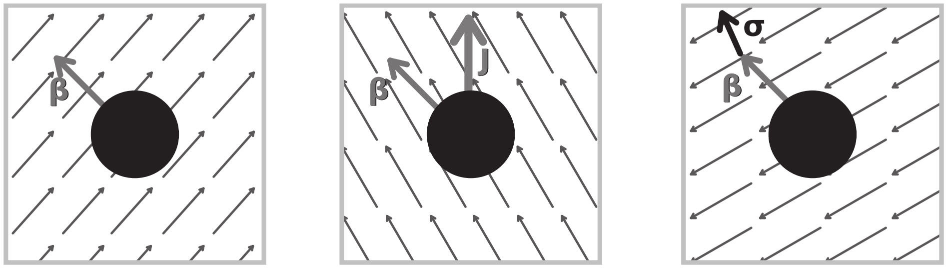

Figure 2.1: Correlations in β decay allowing for various modes of LIV. There can be a correlation between a preferred direction $\hat{\boldsymbol{N}}$, indicated by the background arrows, and the β momentum, nuclear spin or β spin, respectively.

Here the SM double correlations can be used as the normalization for the search of LIV. The triple correlation (Q) is too complicated for this purpose. The normalization can be obtained from double correlations only.

Because the dependence on $\boldsymbol{\beta}_e = \boldsymbol{p}_e/E_e$ was already addressed before, albeit for forbidden decays, it was chosen to pursue a measurement of the dependence on spin direction $\langle \boldsymbol{J} \rangle / J$, while in parallel the theory would be further developed. To measure the dependence on β spin would be too complicated, since it would require a directionally-dependent β scattering experiment [18]. In hindsight, the theory developed also does not justify such a measurement. According to theory discussed below $\boldsymbol{\xi}_3 \propto \boldsymbol{\xi}_1$. Because in Fermi (F) decays the parent spin is a "spectator", i.e. the combined spin of β and neutrino is zero, the LIV coefficient $\boldsymbol{\xi}_2$ is probably best measured in a Gamow-Teller (GT) transition, where the spin of β and neutrino add and there is a correlation with the nuclear spin [Eq. (2.1)]. This aspect is corroborated by theory. In addition the nuclear polarization, i.e. $\langle \boldsymbol{J} \rangle / J$ can be measured via the β asymmetry. The corresponding parameter A is only non-zero for a non-zero GT matrix element in the β transition.

2.2 Theoretical framework

At the university of Groningen a theoretical framework for LIV in the weak interaction was developed [9, 10, 19, 20, 21, 22, 23]. LIV is described in an effective field theory. The weak interaction is mediated by the W and Z bosons. Modifications of SM β decay due to LIV can occur at the initial and final states, the interaction vertices or the boson propagator. For the nucleon, electron and positron, most parameters leading to modification of the initial and final states are more effectively probed by tests in the EM interaction. Also, it was shown that propagator and vertex corrections can be parametrized in the same way, except for a possible dependence on the W boson or quark momenta [9]. Therefore, only the propagator is considered.

The derivation of the expression starts with the SM description and adds a general Lorentz-invariance violating tensor $\chi^{\mu\nu}$ to the W-boson propagator. The W-boson propagator is modified to

$$\langle W^{\mu+}(p)W^{\nu-}(-p)\rangle = \frac{-i\,(g^{\mu\nu}+\chi^{\mu\nu})}{M_W^2}\ . \tag{2.3}$$

Here $g^{\mu\nu}$ is the Minkowski metric, $\chi^{\mu\nu}$ is a general Lorentz-invariance violating tensor, which is complex and possibly momentum-dependent [9], and M_W is the W-boson mass. As a consequence of Eq. (2.3) correlations appear in the decay rate between $\chi^{\mu\nu}$ and (contracted) momenta and spins of the particles involved.

A general expression for the GT transition rate of oriented nuclei was derived in Ref. [9]. The differential decay rate is given by

$$\begin{aligned}
&\omega_{GT} dE_e d\Omega_e \\
&\quad = \omega_0 dE_e d\Omega_e \left\{ \left[1 - \tfrac{2}{3}\chi_r^{00} + \tfrac{2}{3}(\chi_r^{l0} + \tilde{\chi}_i^l)\frac{p_e^l}{E_e} \right] \right. \\
&\qquad \mp \Lambda^{(1)} \left[(1-\chi_r^{00})\frac{\boldsymbol{p}_e \cdot \hat{I}}{E_e} + \tilde{\chi}_i^l \hat{I}^l + \frac{\chi_r^{lk} p^l \hat{I}^k}{E_e} - \frac{\chi_i^{l0}(\boldsymbol{p}_e \times \hat{I})^l}{E_e} \right] \\
&\qquad + \Lambda^{(2)} \left[-\chi_r^{00} + (\chi_r^{l0} + \tilde{\chi}_i^l)\frac{p_e^l}{E_e} + 3\chi_r^{kl}\hat{I}^k\hat{I}^l - 3\chi_r^{l0}\hat{I}^l \frac{\boldsymbol{p}_e \cdot \hat{I}}{E_e} \right. \\
&\qquad\qquad \left. \left. -3\chi_i^{ml}\hat{I}^m \frac{(\boldsymbol{p}\times\hat{I})^l}{E_e} \right] \right\}\ .
\end{aligned} \tag{2.4}$$

We write the polarization vector of the parent nucleus $\boldsymbol{I} = \langle \boldsymbol{J}\rangle / J$ so that $\hat{I}$ in Eq. (2.4) is a unit vector parallel to the nuclear polarization axis of the parent nucleus; $\chi_{r,i}$ are the real and imaginary part of χ, respectively;

ξ_μ	F	GT
ξ_0	$2\chi_r^{00}$	$-\frac{2}{3}\chi_r^{00}$
ξ_1^l	$2\chi_r^{0l}$	$\frac{2}{3}(\chi_r^{l0} + \epsilon_{lmk}\chi_i^{mk})$
ξ_2^l	n.a.	$A\epsilon_{lmk}\chi_i^{mk}$
ξ_3^l	$\mp\sqrt{(1-(\alpha Z)^2)(1-\beta^2)}\xi_1^l$	$\mp\sqrt{(1-(\alpha Z)^2)(1-\beta^2)}\xi_1^l$

Table 2.1: Expressions for ξ_0 and $\boldsymbol{\xi}_{1,2,3}$ in terms of $\chi^{\mu\nu}$. k, l, m refer to the spatial indices.

$\tilde{\chi}^l \equiv \epsilon^{lmn}\chi^{mn}$ with ϵ^{lmn} the Levi-Civita symbol. We have

$$\omega_0 = \frac{1}{2\pi^5}|\boldsymbol{p}_e|E_e(E_e - E_0)^2 F(E_e, \pm Z)\bar{\xi} \tag{2.5}$$

with $F(E_e, \pm Z)$ the Fermi function and $\bar{\xi} = 2(C_V^2 \langle 1\rangle^2 + C_A^2 \langle \boldsymbol{\sigma}\rangle^2)$; $\langle 1\rangle$ and $\langle \boldsymbol{\sigma}\rangle$ are the transition matrix elements of the nucleus for vector and axial-vector interaction, respectively; C_A and C_V are constants for the relative magnitude of vector and axial-vector interaction, respectively. In GT transitions $\langle 1\rangle = 0$. $\Lambda^{(1)}$ is the first-order anisotropy *i.e.* the polarization dependence, calculated from the matrix element. For GT β decay the term simplifies to

$$\Lambda^{(1)} = AP \tag{2.6}$$

with $P = |\boldsymbol{I}|$ the polarization and A the β-asymmetry parameter given by

$$A = \begin{cases} 1 & \text{if } J \rightarrow J-1 \\ \frac{1}{J+1} & \text{if } J \rightarrow J \\ -\frac{J}{J+1} & \text{if } J \rightarrow J+1 \end{cases} . \tag{2.7}$$

$\Lambda^{(2)}$ is the second order anisotropy *i.e.* the alignment, which is even in nuclear spin and is not measured in this experiment. The alignment associated with LIV is not considered in this thesis: all χ terms can be measured using other parts of Eq. (2.4).

The general expression derived for the decay rate confirmed the general form of Eq. (2.2) [9]. The correspondence between the terms with ξ_0 and $\boldsymbol{\xi}_{1,2,3}$ in Eq. (2.2) and the terms in the χ tensor framework is given in Table 2.1. Expressions of χ for forbidden transitions are given in Ref. [10].

We briefly treat the relation between $\chi^{\mu\nu}$ and the parameters of the SME [8]. For the minimal SME, containing all terms up to mass dimension four, in the low-energy approximation we have

$$\chi^{\mu\nu} = -k_{\phi\phi}^{\mu\nu} - \frac{i}{2g} k_{\phi W}^{\mu\nu} + \frac{2p_\rho p_\sigma}{M_W^2} k_W^{\rho\mu\sigma\nu} \,, \tag{2.8}$$

where $k_{\phi\phi}^{\mu\nu}$, $k_{\phi W}^{\mu\nu}$ and $k_W^{\rho\mu\sigma\nu}$ are SME parameters as defined in [11], and $g^2/(8M_W^2) = G_F/\sqrt{2} \approx 0.65$ is the SU(2) coupling constant, with G_F the Fermi coupling constant; $k_{\phi\phi}^{\mu\nu}$ and $k_W^{\rho\mu\sigma\nu}$ have a real and imaginary part and $k_{\phi W}^{\mu\nu}$ only has a real part. In principle, LIV terms of mass-dimension higher than four also give a contribution to the propagator, but these contributions can also be parametrized by $\chi^{\mu\nu}$.

From Table 2.1 we read that $\xi_2^k = A\epsilon_{lmk}\chi_i^{mk}$. This confirms that $\boldsymbol{\xi}_2$ can be interpreted as a preferred direction in absolute space. $\boldsymbol{\xi}_2$ is fixed in absolute space, and would lead to a sinusoidal variation of the term $\boldsymbol{\xi}_2 \cdot \langle \boldsymbol{J} \rangle / J$ in the laboratory, with a period of a sidereal day ($T_{\text{sid.}} = 23^h 56^m$).

2.3 Current status of χ

The best bounds on some of the elements of χ are of the order 10^{-6} to 10^{-8}. They have been obtained in the analysis of Ref. [10], were data from the experiments of Refs. [13, 14] were used. In these experiments the dependence of the decay rate on the direction of the β particle in forbidden transitions was measured. This is sensitive to combinations $\chi_r^{l0} + \tilde{\chi}_i^l$ [Eq. (2.4)], therefore the bounds may be weaker if cancellations occur. Our experiment is exclusively sensitive to $\tilde{\chi}_i^l$. A complete discussion of all experimental bounds on χ is given in Section 5.1.

2.4 Detection strategy and isotope selection

We noted (Table 2.1) that a GT (or mixed) transition is required to measure nuclear-spin-dependent LIV. For nuclear polarization generated by laser light an alkali metal is preferred, since polarizing its nucleus in a buffer gas using circularly polarized light typically requires only light from one laser. The required intensity and wavelength of the laser light set practical limits. As a preference the laser power should be sufficient to reach the saturation intensity and the wavelength should be in the visible or near infrared region. This limits the accessible isotopes to those of the alkali elements Li, Na, K, Rb, Cs [24] and Fr [25].

The experiment was designed to measure the decay rate from the emission of γ photons from the daughter nucleus, rather than from β particles directly. For β particles, the SM parity violation would give a large polarization dependence which would be hard to separate from the LIV contribution. In contrast, the γ ray emission does not depend on the sign of the polarization and thus measures the alignment. Therefore the decay is inferred from the γ ray emitted by the daughter nucleus. It also enables the use of detectors with a large solid angle and consequently higher efficiency. If $R^{\pm}$ is the γ decay rate measured for nuclear polarization in the $\pm\hat{I}^l$ direction, then it follows from Eqs. (2.2) and (2.4) that

$$A_\tau = \frac{R^+ - R^-}{R^+ + R^-} = P\xi_2^l = AP\tilde{\chi}_i^l \ . \tag{2.9}$$

The polarization is determined from the β asymmetry A_β. From Eq. (2.2) it follows that

$$A_\beta = \frac{1}{K}\frac{R_\beta^+ - R_\beta^-}{R_\beta^+ + R_\beta^-} = AP \ . \tag{2.10}$$

$R_\beta^{\pm}$ is the β particle rate measured with a detector positioned on the nuclear polarization axis in the direction $\pm\hat{I}$. K is the analyzing power. It follows that

$$\xi_2^l = \frac{A_\tau}{P} \tag{2.11}$$

and

$$\tilde{\chi}_i^l = \frac{A_\tau}{A_\beta} \ . \tag{2.12}$$

This is valid for mixed transitions as well [9], where A will be reduced compared to a pure GT transition.

The isotope should be selected for a minimal statistical measurement uncertainty σ_{stat}. It is inversely proportional to several factors,

$$\sigma_{\text{stat}}^{-1} \propto |AP\cos\theta| \sqrt{TR_\gamma f} = |AP\cos\theta| \sqrt{T\bar{R}_\beta Br f n_\gamma} \ . \tag{2.13}$$

isotope	J_i^P	J_f^P	Br (%)	A	lifetime (s)	E_γ (keV)
^{20}Na	2^+	2^+	79	$1/3$	0.448	1633
^{25}Na	$\frac{5}{2}^+$	$\frac{3}{2}^+$	27	1	59.1	975 and 390 + 585
^{35}K	$\frac{3}{2}^+\star\star$	$\frac{3}{2}^+$	36	$2/5\star$	0.178	$2590 + 2983$
^{36}K	2^+	2^+	42	$1/3\star$	0.341	$2433 + 2208 + 1970$
^{47}K	$\frac{1}{2}^+$	$\frac{1}{2}^+$	80	$2/3\star$	17.5	$586 + 2014$
	$\frac{1}{2}^+$	$\frac{3}{2}^+$	19	$-1/3\star$	17.5	$565 + 2014$
^{80}Rb	1^+	2^+	22	$-1/2$	34.4	617
^{82}Rb	1^+	2^+	14	$-1/2$	75.5	777

Table 2.2: Decay properties of selected alkali isotopes. $\star$ indicates that these isotopes may have a mixed transition and/or additional decay channels that may dilute the effectively measured A. $\star\star$ indicates the property is not measured. The last column indicates the energies of γ rays of the EM decay (chain) most frequent after the β decay.

Here θ is the angle between the polarization axis and Earth's equatorial plane. T is the measurement time. R_γ is the γ-ray rate for the selected decay channel. The detection efficiency f depends primarily on the angular coverage, attenuating materials, detection method and detector materials. $\bar{R}_\beta$ is the β-decay rate inside the fiducial volume. Br is the branching fraction of the selected β decay. n_γ is the number of photons emitted per selected β decay. θ is the angle between the nuclear polarization axis and the equatorial plane.

In our experiment $\cos\theta \approx 1$ (see Section 4.2). To maximize $\sigma_{\text{stat}}^{-1}$ a branching fraction close to unity is preferred. In addition, in the first experiment, it was found that the polarization lifetime of ^{20}Na was limited to about one second. For a high value of P the lifetime of the selected isotope should be comparable or smaller than the polarization lifetime. The latter depends on the chemical processes and the typical diffusion time in the polarization volume. In our selection we excluded lifetimes larger than two minutes.

With the above considerations suitable isotopes are ^{20}Na, ^{25}Na, ^{35}K, ^{36}K, ^{47}K, ^{80}Rb and ^{82}Rb. Their decay properties are given in Table 2.2. Although ^{47}K has the highest factor $A^2 Br\, n_\gamma$, it has a relatively long lifetime. Moreover, ^{47}K may have a mixed transition and/or additional decay channels that dilute the effectively measured A. ^{20}Na has a relatively small lifetime and the second-highest $A^2 Br\, n_\gamma$. Therefore, ^{20}Na is the best candidate. It is also a pure GT emitter. With the TRIμP facility [26] ^{20}Na was produced for a variety of experiments with intensities $\bar{R}_\beta$ of approximately 10^6 /s as a secondary beam. Moreover, a solid-state laser system suitable for polarizing Na was available.

2.5 Polarization

^{20}Na atoms were polarized by optical pumping in a buffer gas in a weak magnetic field [27]. For this they were irradiated with circularly polarized light (σ^+ or σ^-) tuned to the D_1 transition of ^{20}Na. The energy levels arising from Coulomb interaction are split due to fine structure (spin-orbit coupling), hyperfine structure (coupling of electronic and nuclear magnetism) and the Zeeman effect (coupling with external magnetic field). The contribution of the Zeeman effect depends on the strength of an external magnetic field. The external magnetic field is typically used to define the quantization axis.

In the absorption of σ^+ or σ^- light one unit $\hbar$ of angular momentum is transferred. In this process the magnetic quantum number m_F changes by one unit, *i*.e. $\Delta m_F = +1$ for σ^+ light and $\Delta m_F = -1$ for σ^- light. For the decay of the atom to the ground state with a single photon emission we can have $\Delta m_F = -1, 0, +1$. Therefore, there is a net shift to the highest (lowest) m_F states for σ^+ (σ^-) light. The nuclear spin is coupled to the electronic spin by the hyperfine interaction. The maximum (minimum) in the m_F distribution thus corresponds to the maximal (minimal) possible nuclear polarization. By collision with the gas molecules, the polarization of the atom is continuously destroyed. The details of the processes taking place are not very well known and therefore not fully quantifiable. The polarization buildup has a typical timescale of 1 ms. The polarization is destroyed by collisions on a timescale of 100 ms. Thus we can obtain a net polarization for the whole sample. More details are discussed in Chapter 3.